CS4641B Machine Learning

Focus video: Clustering evaluation – entropy based measures

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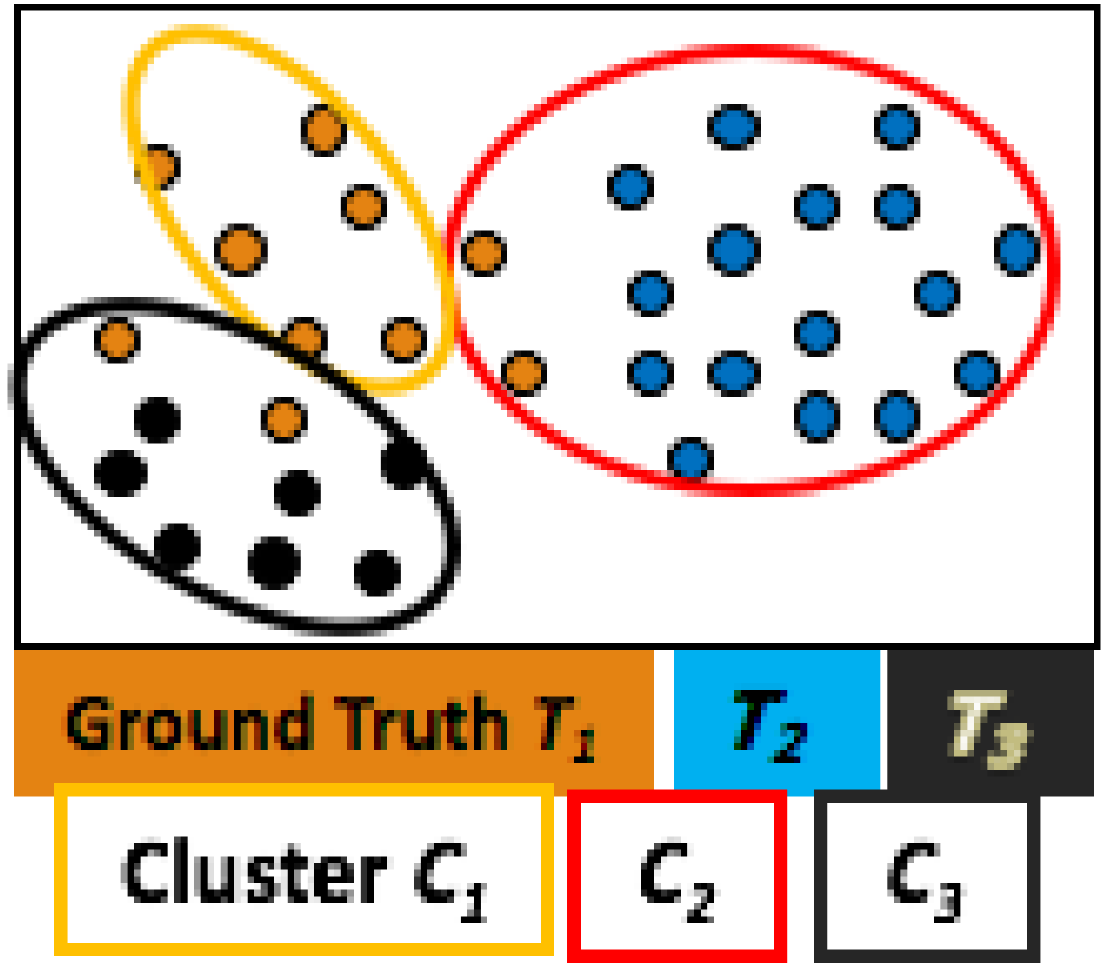
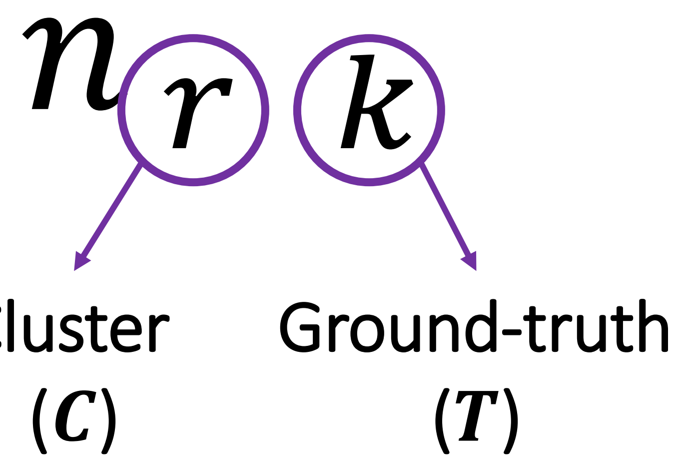
Entropy-based measures: example



C = cluster

T = partition

	$k=1$	$k=2$	$k=3$	n_r
$r=1$	$n_{11}=6$	$n_{12}=0$	$n_{13}=0$	6
$r=2$	$n_{21}=2$	$n_{22}=16$	$n_{23}=0$	18
$r=3$	$n_{31}=2$	$n_{32}=0$	$n_{33}=7$	9
m_k	10	16	7	33



Joint probability distribution

$p(C = r, T = k)$

T = partition

C = cluster

	$k = 1$	$k = 2$	$k = 3$	p_{C_r}
$r = 1$	$\frac{6}{33}$	$\frac{0}{33}$	0	$\frac{6}{33}$
$r = 2$	$\frac{2}{33}$	$\frac{16}{33}$	0	$\frac{18}{33}$
$r = 3$	$\frac{2}{33}$	$\frac{0}{33}$	$\frac{7}{33}$	$\frac{9}{33}$
p_{T_k}	$\frac{10}{33}$	$\frac{16}{33}$	$\frac{7}{33}$	1

$p(r = 2)$

$p(k = 2 | r = 3)$

Joint probability distribution

$p(C = r, T = k)$

T = partition

C = cluster

	$k = 1$	$k = 2$	$k = 3$	p_{C_r}
$r = 1$	0.182	0.0	0.0	0.182
$r = 2$	0.060	0.485	0.0	0.545
$r = 3$	0.060	0.0	0.212	0.272
p_{T_k}	0.303	0.485	0.212	1

Entropy

- For clusters (result from clustering)

$$H(\mathcal{C}) = -\sum_{r=1}^{R} p_{C_r} \log_2 p_{C_r} = p_{C_1} \log_2 p_{C_1} + p_{C_2} \log_2 p_{C_2} + p_{C_3} \log_2 p_{C_3}$$

$$H(\mathcal{C}) = -0.182 \log_2 0.182 - 0.545 \log_2 0.545 - 0.272 \log_2 0.272$$

$$H(\mathcal{C}) = 1.435\ bits$$

- For partitions (ground-truth)

$$H(\mathcal{T}) = -\sum_{r=1}^{R} p_{T_k} \log_2 p_{T_k} = p_{T_1} \log_2 p_{T_1} + p_{T_2} \log_2 p_{T_2} + p_{T_3} \log_2 p_{T_3}$$

$$H(\mathcal{T}) = -0.303 \log_2 0.303 - 0.485 \log_2 0.485 - 0.212 \log_2 0.212$$

$$H(\mathcal{T}) = 1.503\ bits$$

Conditional probability distribution

$$p(T = k | C = r)$$

T = partition

C = cluster

	$k = 1$	$k = 2$	$k = 3$
$r = 1$	$\frac{6}{6}$	$\frac{0}{6}$	$\frac{0}{6}$
$r = 2$	$\frac{2}{18}$	$\frac{16}{18}$	$\frac{0}{18}$
$r = 3$	$\frac{2}{9}$	$\frac{0}{9}$	$\frac{7}{9}$

$p(k = 3 | r = 2)$

$p(k = 2 | r = 3)$

Conditional probability distribution

$p(T = k|C = r)$

T = partition

C = cluster

	$k = 1$	$k = 2$	$k = 3$
$r = 1$	1.0	0	0
$r = 2$	0.111	0.889	0
$r = 3$	0.222	0	0.778

Conditional entropy

- For each cluster:

$$H(\mathcal{T}|C_1) = -1.0 \log_2 1.0 - 0.0 \log_2 0.0 - 0.0 \log_2 0.0 = 0.0\ bits$$

$$H(\mathcal{T}|C_2) = -0.111 \log_2 0.111 - 0.889 \log_2 0.889 - 0.0 \log_2 0.0 = 0.503\ bits$$

$$H(\mathcal{T}|C_3) = -0.222 \log_2 0.222 - 0.0 \log_2 0.0 - 0.778 \log_2 0.778 = 0.764\ bits$$

- Conditional entropy

$$H(\mathcal{T}|\mathcal{C}) = \sum_{R} P(C_r) \times H(\mathcal{T}|C_r) = p_{C_1} \times H(\mathcal{T}|C_1) + p_{C_2} \times H(\mathcal{T}|C_2) + p_{C_3} \times H(\mathcal{T}|C_3)$$

$$H(\mathcal{T}|\mathcal{C}) = 0.182 \times 0.0 + 0.545 \times 0.503 + 0.272 \times 0.764 = 0.482\ bits$$

Normalized mutual information

- Mutual information

$$I(\mathcal{C},\mathcal{T}) = H(\mathcal{T}) - H(\mathcal{T}|\mathcal{C}) = 1.503 - 0.482 = 1.021\ bits$$

- Normalized mutual information

$$NMI(\mathcal{C},\mathcal{T}) = \frac{I(\mathcal{C},\mathcal{T})}{\sqrt{H(\mathcal{C}) \times H(\mathcal{T})}} = \frac{1.021}{\sqrt{1.503 \times 1.435}} = 0.695\ bits$$