CS4641B Machine Learning

Focus video: MLE

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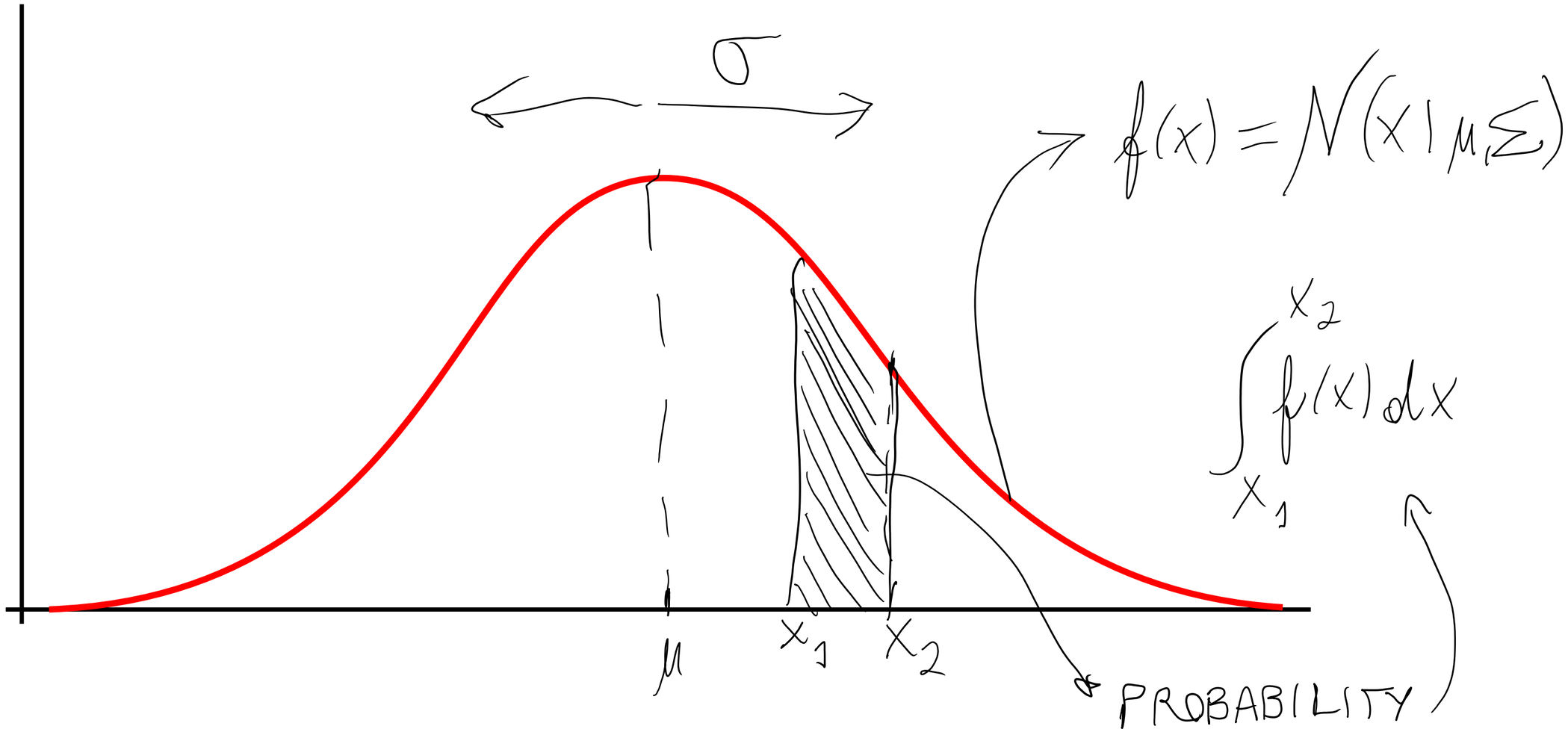
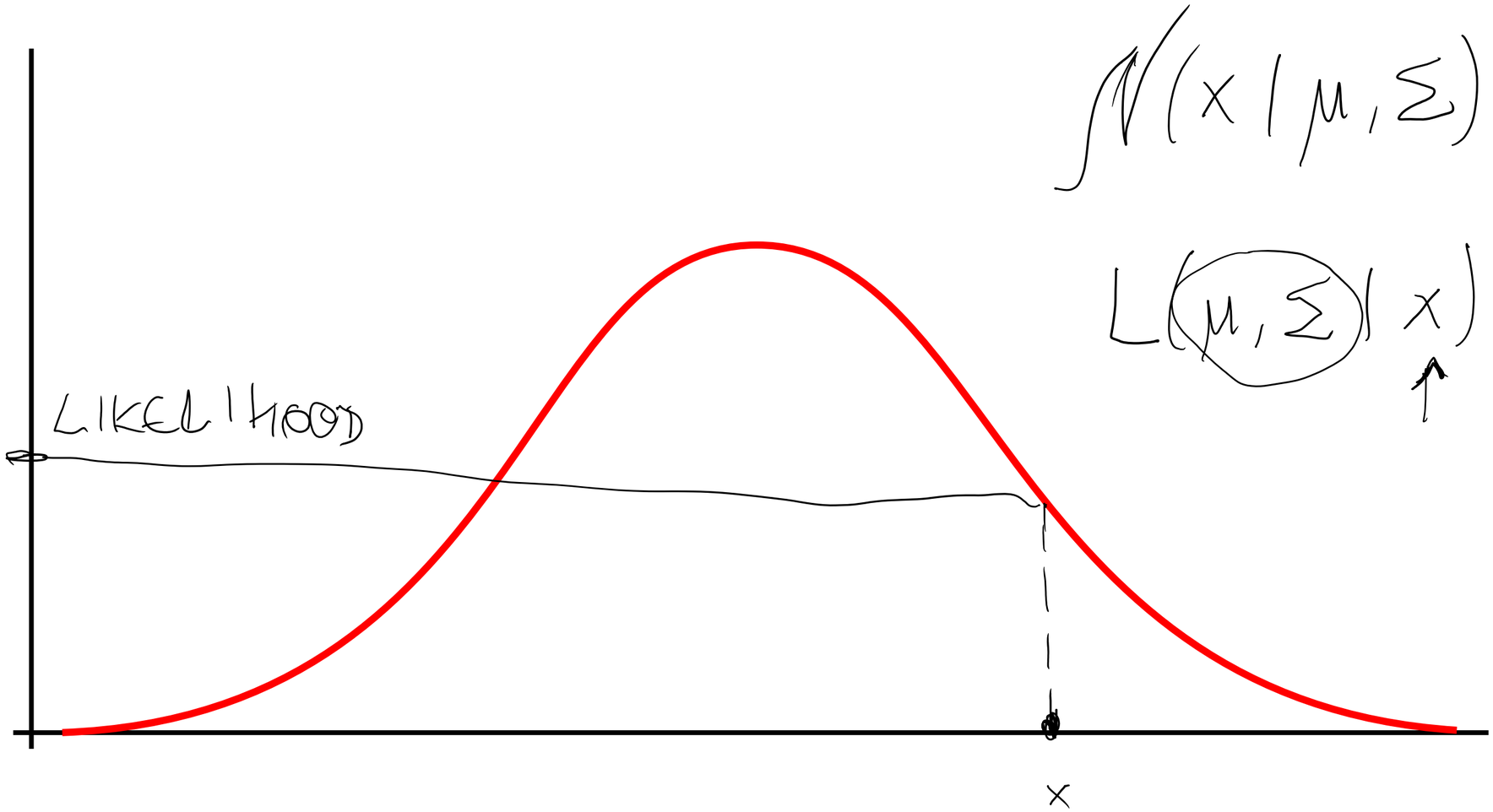
Likelihood vs probability



Likelihood vs probability



Maximum likelihood estimation

Exponential distribution

Probability density: $p(x|\mu) = \frac{1}{\mu} e^{-\frac{x}{\mu}}, \text{for } x \geq 0$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$$L(\mu | x_1) = \frac{1}{\mu} \exp\left(-x_1/\mu\right)$$

$$\frac{dL}{d\mu} = 0 \longrightarrow \text{FIND OPT. } \mu$$

Maximum likelihood estimation

$$L(\mu \mid x_1, x_2, x_3, \cdots, x_n) \xrightarrow{i.i.d} L(\mu \mid x_1) L(\mu \mid x_2) \cdots L(\mu \mid x_n)$$

$$L(\mu \mid X) = \prod_i^n \frac{1}{\mu} \exp\left(-\frac{x_i}{\mu}\right)$$

$$\log L = \sum_i^n \underbrace{\log\left(\frac{1}{\mu}\right)}_{c} + \sum_i^n \log\left(\exp\left(-\frac{x_i}{\mu}\right)\right)$$

$$\log L = n \log(1/\mu) - \sum_i^n \left(\frac{x_i}{\mu}\right) \qquad 1/\mu = \lambda$$

Maximum likelihood estimation

$$LL = m \log(\lambda) - \lambda \sum_{i}^{m} x_i$$

OPTIMIZE λ

$$\frac{\partial LL}{\partial \lambda} = 0 \rightarrow \frac{1}{\lambda} m - \sum_{i}^{m} x_i = 0$$

$$\hat{\lambda} = \frac{m}{\sum_{i}^{m} x_i}$$